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Are Bound States of Color-Excited Leptons Responsible for Anomalous e^+e^- Production in Heavy Ion Collisions?

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A model for the anomalous e^+e^- production in heavy ion collisions is proposed. The model is based on the hypothesis that e^+e^- production derives from the decay of "leptopions," pionlike bound states of colored excitations of e^+ and e^-. Order-of-magnitude estimates for the mass scale of the radial excitations and lifetime of the leptopion obtained by extrapolation from the case of the ordinary pion are in accordance with data. The model for leptopion production is based on the electromagnetic anomaly term. In the classical treatment of the nucleus-nucleus collision the leptopion production amplitude is essentially the Fourier transform of the scalar product of the electric field of the stationary target nucleus and the magnetic field of the colliding nucleus. The production amplitude becomes singular for certain values of the kinematical variables and in singularity the velocity of the leptopion is a definite function of the production angle measured with respect to the direction of the collision velocity. Due to the weak dependence of the velocity of the production angle in the forward direction the leptopions are apparently produced at rest in center-of-mass coordinates in accordance with the data. The observed peak of e^+e^- production amplitude (in fact two peaks in some cases) is explained as a quantum diffraction effect resulting from the finite size of the colliding nuclei, when the collision velocity exceeds the velocity needed to overcome the Coulomb barrier. The production amplitude oscillates as a function of collision velocity and the period of oscillation is in accordance with the width of the observed velocity peak. In principle, several diffraction peaks are possible for elastic collisions in a plane, but the effects of strong interactions are expected to lead to the disappearence of the peaks at velocities larger than that needed to overcome the Coulomb barrier. An explanation for the unobservability of lepton color via strong interactions is proposed.

1. HEAVY-ION COLLISION EXPERIMENTS

Heavy ion-collision experiments carried out at the Gesellschaft fur Schwerionenforschung in Darmstadt, West Germany (Schweppe *et al.*, 1983;

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Clemente *et al.*, 1984; Cowan *et al.*, 1985, 1986; Tsertos *et al.*, 1985, 1987) have yielded a rather puzzling set of results. The expectation was that in heavy ion collisions in which the combined charge of the two colliding ions exceeds 173, a composite nucleus with $Z > Z_{cr}$ would form and the probability for spontaneous positron emission would become appreciable.

Indeed, narrow peaks of widths of roughly 50–70 keV and energies about 350 ± 50 keV were observed in the positron spectra, but it turned out that the position of the peaks seems to be a constant function of Z rather than vary as Z^{20} as expected and that peaks are generated also for Z smaller than the critical Z. The collision energies at which peaks occur lie in the neighborhood of 5.7–6 MeV/nucleon. Also, it was found that positrons are accompanied by e emission. Data are consistent with the assumption that some structure at rest in the center-of-mass system is formed and decays subsequently to a e^+e^- pair.

Various theoretical explanations for these peaks have been suggested (Chodos, 1987; Kraus and Zeller, 1986). For example, lines might be created by pair conversion in the presence of heavy nuclei. In nuclear physics explanations the lines are due to some nuclear transition that occurs in the compound nucleus formed in the collision or in the fragments formed. The Z independence of the peaks seems, however, to exclude both atomic and nuclear physics explanations (Pitkänen, 1981, 1983, 1985, 1986*a,b*, 1988). Elementary particle physics explanations (Chodos, 1987; Kraus and Zeller, 1986) seem to be excluded already by the fact that several peaks have been observed, one at 1.062 MeV and possibly several peaks in the range 1.6–1.8 MeV.

Thus, it seems that the structures produced might be composite, perhaps resonances in the e^+e^- system. The difficulty of this explanation is that conventional QED seems to offer no natural explanation for the strong force needed to explain the energy scale of the states. One idea is that the strong electromagnetic fields create a new phase of QED (Chodos, 1987) and that the resonances are analogous to pseudoscalar mesons appearing as resonances in strongly interacting systems.

The explanation proposed in this paper is based on the following hypotheses motivated by topological geometrodynamics (Pitkänen, 1981, 1983, 1985, 1986*a,b*; 1988):

(a) Ordinary leptons are nonpointlike particles and can have colored excitations.

(b) e^+e^- structures are "leptopions," color-confined states formed from the colored excitations of e^+ and e^-.

In the following I shall:

(a) Demonstrate that one can qualitatively understand the masses and lifetimes of the observed states using these hypotheses.

(b) Construct a model for the leptopion production treating nucleus-nucleus collision purely classically and assuming that leptopions are produced through an electromagnetic anomaly term (Itzykson and Zuber, 1980) so that the production amplitude is essentially the Fourier transform of the scalar product of the electric field of the stationary target nucleus with the magnetic field of the colliding nucleus. This model explains why leptopions are apparently produced at rest in cm coordinates. This model does not, however, explain the peak structure observed in leptopion production.

(c) Modify the model by taking into account the quantum diffraction effects resulting from the finite size of the nuclei. As a result I obtain an explanation for the velocity peak(s). The production amplitude is predicted to contain an oscillatory term above the velocities needed to overcome the Coulomb barrier in the case of elastic plane collisions. The effects of strong interactions are expected to lead to the disappearence of the oscillatory behavior at higher energies.

(d) Describe briefly the topological geometro dynamics (TGD) approach (Pitkänen, 1981, 1983, 1985, 1986*a*,*b*, 1988), which has provided the general philosophy behind the colored lepton hypothesis.

(e) Propose a TGD-based explanation for the absence of lepton-hadron color interactions.

2. ESTIMATES FOR THE LIFETIME OF THE LEPTOPION AND FOR THE MASSES OF THE EXCITATIONS OF LEPTOPIONS

The assumption that leptopions are color-confined states of color octet fermions makes it possible to estimate the masses and lifetimes for leptopion states.

A string picture of leptopions is suggested by analogy with ordinary pions and an estimate for the string tension is obtained by multiplying the hadronic string tension 1 GeV^2 by the ratio of masses for color octed lepton and for the u-quark and by the ratio $r=2$ of Casimir operators in octet and triplet representations,

$$\begin{aligned} T_L/T_H &= k(m_{e1}/m_u) \\ k &= (m_{e1}/m_u)C(8)/C(3) \simeq 1.7\times 10^{-3} \end{aligned} \tag{1}$$

I assume the value $m_u = 350$ MeV for the quark mass (m_{e1} denotes the mass of the colored lepton).

An estimate for the mass of the color octet lepton is obtained by requiring the string tension to be such that the state with mass 1.6 MeV (radial excitation of the lowest-lying state in the "leptonium" picture) corresponds to the lowest-lying state in the daughter trajectory of the Regge trajectory associated with the 1.062-MeV state, so that the mass of this state should be approximately given by the expression

$$m = (m_0^2 + nT_L)^{1/2}$$
$$m_0 \simeq 1 \text{ MeV} \tag{2}$$

with $n = 1$.

The resulting value of the string tension is $T_L \simeq 1.4 \text{ MeV}^2$ and the value of the mass of the colored lepton is $m \simeq 0.3$ MeV. The mass of the $n = 2$ state is estimated to be 1.9 MeV, to be compared with the value 1.8 MeV (of course, the situation changes if there exist additional resonances in the mass range 1.6–1.8 MeV).

An estimate for the mass splitting between leptopion and "leptorho" ($L = 0$ state with parallel spins) is obtained from ρ–π mass splitting by multiplying it with the ratio $r = 2$ of the values of the Casimir operator in the octet and triplet representations and with the mass ratio m_{e1}/m_u:

$$\rho_L - \pi_L = (\rho - \pi)k \tag{3}$$

The splitting obtained is 1.2 MeV and the mass of leptorho is predicted to be 2.2 MeV.

A leptopion with a mass 1.062 MeV decays mainly into two photons via an anomaly term (Itzykson and Zuber, 1980). The ratio for the lifetimes of the leptopion and the pion is thus expected to be given by

$$t_L = t_\pi / k \tag{4}$$

and one obtains the estimate $t_L = 4.0 \times 10^{-14}$ sec for the lifetime of the leptopion. This estimate is consistent with the experimental constraints on the lifetime ($10^{-9} > t_L > 10^{-19}$ sec).

These rough estimates clearly show that the present hypothesis is in accordance with the existing data concerning the mass scale associated with the e^+e^- states and the lifetime of the lowest e^+e^- state.

3. MODEL FOR THE PRODUCTION OF LEPTOPIONS TREATING NUCLEI AS CLASSICAL POINTLIKE CHARGES

The angular momentum barrier makes the production of leptomesons with orbital angular momentum $L > 0$ unprobable. Therefore the observed resonances are expected to be $L = 0$ pseudoscalar states. Leptopion production has two signatures which any realistic model should reproduce.

(a) Data are consistent with the assumption that states are produced at rest in the cm frame.

(b) The production probability has a peak in a narrow region of velocities of the colliding nucleus around the velocity needed to overcome the Coulomb barrier in a head-on collision. The relative width of the velocity peak is of order $\Delta\beta/\beta \simeq 10^{-2}$.

In this section a model treating nuclei as pointlike charges and nucleus–nucleus collision purely classically is developed. This model yields predictions in close agreement with the signature (a), but fails to reproduce signature (b).

The basic ingredients for the classical model of leptopion production are the following:

(a) The well-known relation (Itzykson and Zuber, 1980) expressing the pion field as a sum of the divergence of an axial vector current and an electromagnetic anomaly term generalizes to the case of the leptopion as

$$\pi = [\nabla \cdot j^A + (\alpha_{em}/8\pi)E \cdot B]/m_0^2 f \tag{5}$$

In the case of the pion field, $f_\pi = 93$ MeV has the order of magnitude of the pion mass. In the leptopion case the order of magnitude of f is given by

$$f = kf_\pi \simeq 0.16 \text{ MeV} \tag{6}$$

The anomaly term gives rise to pion decay to two photons, so that one obtains an estimate for the lifetime of the leptopion.

This relation is taken as the basis for the model describing also the production of leptopions in an external electromagnetic field. The idea is that the presence of an external electromagnetic field gives rise to a vacuum expectation value of the leptopion field. Vacuum expectation is obtained by assuming that the vacuum expectation value of the axial vector current vanishes,

$$\begin{aligned} \langle \text{vac}|\pi|\text{vac}\rangle &= KE \cdot B \\ K &= \alpha_{em}/(8\pi f m_0^2) \end{aligned} \tag{7}$$

(b) The amplitude for the production of leptopions with four-momentum $p = (p_0, \bar{p})$ is an external electromagnetic field is obtained by substituting the expression for the vacuum expectation of the leptopion field into the LSZ reduction formula (Itzykson and Zuber, 1980)

$$\begin{aligned} A(p) &= \int f_p(x)\, \Box \langle \text{vac}|\pi|\text{vac}\rangle\, d^4x \\ f_p &= e^{ip\cdot x}/(4\pi p_0 V)^{1/2} \end{aligned} \tag{8}$$

Here V denotes the quantization volume.

The probability for the production of leptopions in a phase space volume element d^3p is obtained by multiplying with the density of states factor Vd^3p,

$$dP = A|U|^2\, d^3p$$
$$A = (\alpha_{em}/8\pi m_0 f)^2/4\pi p_0 \qquad (9)$$
$$U = \int e^{ip\cdot x} E\cdot B\, d^4x$$

Let us now specialize to the case of a heavy ion collision. Consider the situation where the scattering angle of the colliding nucleus is measured. Treating the collision completely classically, we can assume that collision occurs with a well-defined value of the impact parameter in a fixed scattering plane.

Let us choose the coordinates so that target nucleus is at rest at the origin of the coordinates and the colliding nucleus moves in the z direction in the $y=0$ plane with velocity β. The scattering angle of the scattered nucleus is denoted by α, the velocity of the leptopion by v, and the direction angles of the leptopion velocity by (θ, ϕ).

The minimum value of the impact parameter for the Coulomb collision of pointlike charges is given by

$$b = b_0 \cot(\alpha/2)/2$$
$$b_0 = 2Z_1Z_2\alpha_{em}/M_R\beta^2 \qquad (10)$$

where b_0 is the expression for the distance of closest approach in a head-on collision. M_R denotes the reduced mass of the nucleus–nucleus system.

To evaluate the amplitude for leptopion production, the following simplifying assumptions are made.

(a) Nuclei can be treated as pointlike charges. This assumption is well motivated when the impact parameter of the collision is larger than the critical impact parameter given by the sum of radii of the colliding nuclei:

$$b_{cr} = R_1 + R_2 \qquad (11)$$

For scattering angles that are sufficiently large the values of the impact parameter do not satisfy the above condition in the region of the velocity peak.

(b) Since the velocities are nonrelativistic (about $0.12c$ in the laboratory frame), one can treat the motion of the nuclei nonrelativistically and the retarded electromagnetic fields associated with the exactly known classical orbits can be used. The use of classical orbits does not take into account the recoil effect caused by leptopion production. Since the mass ratio of the leptopion to the reduced mass of the heavy nucleus system is of order 10^{-5}, the recoil effect is, however, negligible.

(c) The model simplifies considerably when the orbit is idealized with a straight line with impact parameter determined from the condition expressing the scattering angle in terms of the impact parameter. This approximation is certainly well founded for large values of the impact parameter.

In this approximation the instanton density in the rest frame of the target nucleus is just the scalar product of the Coulombic electric field E of the target nucleus and of the magnetic field B of the colliding nucleus obtained by boosting it from the Coulomb field of the nucleus at rest.

The Fourier amplitudes of these fields with unit charge are given by the expressions

$$\begin{aligned} E_i(k) &= \delta(k_0)k_i/k^2 \\ B_i(k) &= \delta(\gamma(k_0-\beta k_z))k_j\varepsilon_{ijz}\, e^{ik_x b}/[(k_z/\gamma)^2+k_T^2] \end{aligned} \tag{12}$$

The Fourier component of $E\cdot B$ is given as a convolution of Fourier transforms, which reduces to a two-dimensional integral

$$\begin{aligned} U(p) &= \beta\gamma \int dk_x\, dk_y\, (k_x p_y - k_y p_x)/AB \\ A &= (p_z - p_0/\beta)^2 + p_T^2 + k_T^2 - 2k_T\cdot p_T \\ B &= k_T^2 + (p_0/\beta\gamma)^2 \\ p_T &= (p_x, p_y) \\ k_T &= (k_x, k_y) \end{aligned} \tag{13}$$

One can apply the calculus of residues to calculate the integral with respect to k_y and k_x. As described in the Appendix, one can express the integral over k_x as a sum of residue terms. The integral of the resulting amplitude over k_y can be expressed as a sum of residue contributions plus integrals over two cuts.

The resulting expression for the amplitude reduces to the following general form:

$$U = RES + CUT_1 + CUT_2 \tag{14}$$

The expressions for the various terms are given in the Appendix.

Since my main interest is to show that the production amplitude indeed becomes singular at certain values of kinematical variables, I restrict consideration to the contribution of the first cut, which for $\psi\in[0, \pi/2]$ is given by the expression

$$\mathrm{CUT}_1 = \sin\theta \sin\phi \int_0^{\pi/2} A\, d\psi/2$$

$$A = e^{-bm\gamma_1 \cos\psi/\beta\gamma}(\sin\theta\cos\phi + iK\cos\psi)/X_1$$

$$X_1 = \sin^2\theta(\sin^2\phi - \cos^2\psi) + K^2 - 2iK\sin\theta\cos\psi\cos\phi \tag{15}$$

$$K = \beta\gamma(1 - v_{\mathrm{cm}}\cos\theta/\beta)$$

$$v_{\mathrm{cm}} = 2v/(1+v^2)$$

Using the symmetries

$$A(p_x, -p_y) = -A(p_x, p_y)$$
$$A(-p_x, -p_y) = \bar{A}(p_x, p_y) \tag{16}$$

of the amplitude, one can calculate the amplitude for other values of ϕ.

CUT_1 indeed gives the singular contribution to the amplitude. The reason is that the factor X_1 appearing in denominator of the cut term vanishes when the conditions

$$\cos\theta = \beta/v_{\mathrm{cm}}$$
$$\sin\phi = \cos\psi \tag{17}$$

are satisfied.

In the forward direction this condition says that the z component of the leptopion momentum in the velocity center-of-mass coordinate system vanishes. In the laboratory frame this condition means that the leptopion moves in a certain cone defined by the value of its velocity. The condition is possible to satisfy only above the threshold $v_{\mathrm{cm}} \geq \beta$.

For $K = 0$ the integral reduces to the form

$$\mathrm{CUT}_1 = (\cos\phi\sin\phi/2)\lim_{\varepsilon\to 0}\int_0^{\pi/2} e^{-bm\gamma_1\cos\psi/\beta\gamma}\, d\psi/[\sin^2\phi - \cos^2\psi + i(\varepsilon)] \tag{18}$$

One can estimate the singular part of the integral by replacing the exponent term with its value at the pole. The remaining integrations can be performed using elementary calculus [substituting $t = \tan(\phi/2)$] and one obtains for the singular cut contribution the approximate expression

$$\mathrm{CUT}_1 \simeq e^{-bm\gamma_1\sin\phi/\beta\gamma}\ln(X)/2$$

$$X = [(1+s)^{1/2} + (1-s)^{1/2}]/[(1+s)^{1/2} - (1-s)^{1/2}] \tag{19}$$

$$s = \sin\phi$$

The amplitude diverges logarithmically for $\phi = 0$. In addition, the singular contribution contains an exponential damping term.

If this singular term indeed gives the dominant contribution to the leptopion production, one can draw some conclusions concerning the properties of the production amplitude.

(a) Production occurs mainly in the cone $\cos\theta = \beta/v_{cm}$: in the forward direction this corresponds to the vanishing of the z component of the leptopion momentum in the velocity center-of-mass frame. In addition, the production occurs mainly in the scattering plane due to the dependence of the production amplitude on the angle ϕ (see Figure 3). Numerical calculations and a closer examination of the production amplitude suggest that for all values of θ the production is strongly restricted to the collision plane.

Data are consistent with the production of leptopions at rest in the center-of-mass frame. This would mean an additional restriction $\cos\theta = 0$ to the kinematical variables in the laboratory frame, so that production would occur only on the threshold $v_{cm}/\beta = 1$.

This result need not be in contradiction with the present model. In a singularity the velocity of the leptopion in a given direction θ is a definite function of θ and is given in a good approximation by the expression

$$v = \beta/2\cos\theta \tag{20}$$

If positron and electron pairs are detected in the forward direction, the velocity v depends only weakly on θ. The relative width of the velocity peak is of order $\Delta v/v \simeq 0.2$ (Cowan *et al.*, 1986) and this corresponds to an angular width $\Delta\theta \simeq 34$ deg.

(b) Consider next the velocity dependence of the production amplitude. Substituting the expression of the impact parameter in terms of the collision velocity and scattering angle, we obtain the following expression for the exponent term in the production amplitude:

$$\mathrm{CUT}_1 = \exp[-(Z_1 Z_2 \alpha_{em} m/M_R \beta^3 \gamma)\cot(\alpha/2)\sin\phi]\ln(X) \tag{21}$$

To get a grasp on the situation, consider the U–U collision. In this case, the values of the relevant parameters are estimated to be

$$\begin{aligned} R &\simeq 9.3\times10^{-15}\ \mathrm{m} \\ 1/m &\simeq 1.2\times10^{-12}\ \mathrm{m} \\ \beta &\simeq 0.13 \\ bm &= 2Rm\cot(\alpha/2) \simeq 0.014\cot(\alpha/2) \end{aligned} \tag{22}$$

For physically interesting values of the velocity β (about 0.13) and scattering angle α (20–70 deg) the exponential term is essentially a constant. Only for very small angles is the exponent a rapidly varying function of β.

The conclusion is that a purely classical model based on pointlike nuclei cannot explain the velocity peak, although it makes understandable the peculiar angular distribution of leptopions.

4. INCLUSION OF QUANTUM MECHANICAL AND FINITE-SIZE EFFECTS

The most obvious explanations for the failure of the previous model to predict the velocity peak are the following:

(a) Quantum mechanical effects in the motion of nuclei have been ignored.

(b) The finite size of the colliding nuclei has not been taken into account, although it certainly plays an important role in the collision for the values of the scattering angle and velocity considered.

The physical idea behind the model to be presented is that the velocity peak is a diffraction effect resulting from the quantum mechanical wave nature of the colliding nuclei and from their finite size.

If diffraction effects are important, one expects that the production amplitude contains a factor C which is essentially of the type

$$C = \sin(P_x b_{cr})$$
$$P_x = M_R \beta \sin \alpha \tag{23}$$
$$b_{cr} = R_1 + R_2$$

Here P_x denotes the momentum component of the scattered nucleus in the direction of the impact parameter and b_{cr} is the critical value of the impact parameter given by the sum of the nuclear radii. This quantity varies very rapidly with the collision velocity and changes from 1 to 0 when the relative change of β is of the order of 10^{-2}.

There are two observational indications that the diffraction picture might be correct.

(a) The relative width of the velocity peak is indeed of the order of $\Delta\beta/\beta \simeq 10^{-2}$ (Cowan *et al.*, 1986).

(b) If the diffraction mechanism is the correct explanation for the peak, one expects the presence of the several peaks. Indeed, in the Th–Th system (Cowan *et al.*, 1986) two peaks at projectile energies 5.70 and 5.75 MeV per nucleon have been observed.

I propose the following formulation to take into account quantum mechanical effects related to the finite size of the nuclei. Consider first the quantum mechanical description.

(a) Directions orthogonal to the scattering plane are treated classically and only the coordinate corresponding to the direction of the impact parameter vector is treated quantum mechanically.

(b) To each value of the impact parameter b a different vacuum is associated: the vacuum expectation value of the leptopion field is given as the instanton density of the classical electromagnetic field associated with the classical nucleus-nucleus collision with this value of the impact parameter.

(c) The states of the colliding nucleus are described quantum mechanically as generalized plane waves with momentum given by the projection of the momentum of the colliding nucleus to the direction of the impact parameter vector,

$$|P_x\rangle = \int e^{iP_x b}|b\rangle \otimes |\text{vac}(P, b)\rangle \tag{24}$$

Here b can have arbitrary values and the impact parameter is given as the absolute value of b.

For the incoming nucleus the momentum component P_x vanishes and for the elastically scattered nucleus it is given by the expression

$$P_x = M_R \beta \sin \alpha \tag{25}$$

(d) The amplitude for the leptopion production is obtained as the scalar product of incoming and outgoing states,

$$B(p) = \int e^{iP_x b} A(P, b)\, db \tag{26}$$

Here $A(P, b)$ denotes the amplitude of the purely classical model with fixed impact parameter, essentially the integral of the instanton density over spacetime for the classical collision with incoming momentum P and impact parameter $|b|$.

Consider now the problem of taking into account the finite size of the nuclei.

If the value of the impact parameter is not considerably larger than the sum of the radii of the colliding nuclei, one must use in the collision region realistic charge distributions to calculate their contribution to the production amplitude. Since the value of the leptopion momentum typically corresponds to a de Broglie wavelength of order 10^{-11} m, the phase factor in the plane wave factor appearing in the integrand is essentially constant in this region and we obtain simply the integral of $E \cdot B$ over this region,

$$U(p) \simeq e^{ip \cdot x_{\text{cm}}} \int E \cdot B\, d^4x \tag{27}$$

Here x_{cm} denotes center-of-mass coordinate for the interaction region. $E \cdot B$ reduces to a total divergence and is expressible as an integral over the boundary of the interaction region. Therefore this term does not depend at all on the details of the dynamics in the interaction region: only the boundaries of the interaction region contain the dependence on the dynamics of the collision, in particular on the value of the impact parameter of the collision.

The simplest guess is that the contribution of the interaction region to $U(p)$ vanishes for small values of the impact parameter, since nuclei form a system resembling more closely a single, spherically symmetric nucleus rather than two separate nuclei. The corresponding electromagnetic field is a spherically symmetric Coulomb field and has vanishing instanton density. Since the fields in the spacetime outside the interaction region are weak, one expects that their contribution is small. As a consequence, the instanton term is expected to be vanishing in the lowest order approximation for values of the impact parameter smaller than its critical value.

In this approximation the production amplitude is given by the expression

$$A = A_{\mathrm{id}} - \int_{-b}^{b} e^{iP_x b} A(P, b)\, db \tag{28}$$

A_{id} denotes the amplitude on the limit of pointlike nuclei. In the integral there appears the amplitude $A(P, b)$ associated with pointlike nuclei.

The nice feature of the model is that one can explicitly evaluate the expression for A_{id}. This is due to the fact that $A(P, b)$ is defined as an integral over k_x and k_y of the amplitude, which depends on the impact parameter only through the factor e^{ibk_x}. Substitution of this expression eliminates k_x and b integrations. The k_y integration can be performed analytically (Appendix) and one obtains the following expression for A_{id} as a sum of two terms:

$$A_{\mathrm{id}} = C(U_1 + U_2) \tag{29}$$

Here C is a multiplicative constant derivable from the general expression for the production amplitude.

The explicit expression for the first term is given by

$$\begin{aligned}
U_1 &= \mathrm{RE}_1 + i\mathrm{IM}_1 \\
\mathrm{RE}_1 &= (P_x p_y^2 - p_x\, \mathrm{re}_1/2)/(\mathrm{re}_1^2 + \mathrm{im}_1^2) \\
\mathrm{IM}_1 &= (-P_x p_y\, \mathrm{re}_1/2K_1^{1/2} - p_x p_y K_1^{1/2})/(\mathrm{re}_1^2 + \mathrm{im}_1^2) \\
\mathrm{re}_1 &= (p_z - p_0/\beta)^2 + p_T^2 - (p_0/\beta\gamma)^2 - 2p_x P_x \\
\mathrm{im}_1 &= -2K_1^{1/2} p_y \\
K_1 &= P_x^2 + (p_0/\beta\gamma)^2
\end{aligned} \tag{30}$$

The expression for the second term is given by

$$
\begin{aligned}
U_2 &= \mathrm{RE}_2 + i\,\mathrm{IM}_2\\
\mathrm{RE}_2 &= -[(P_x p_y - p_x P_y)p_y + p_x\, \mathrm{re}_2/2]/(\mathrm{re}_2^2 + \mathrm{im}_2^2)\\
\mathrm{IM}_2 &= [-(P_x p_y - p_x P_y)\, \mathrm{re}_2/2K_2^{1/2} + p_x p_y K_2^{1/2}]/(\mathrm{re}_2^2 + \mathrm{im}_2^2)\\
\mathrm{re}_2 &= -(p_x - p_0/\beta)^2 + (p_0/\beta\gamma)^2 + 2p_x P_x + p_y^2 - p_x^2\\
\mathrm{im}_2 &= 2p_y K_2^{1/2}\\
K_2 &= (p_x - p_0/\beta)^2 + p_x^2 + P_x^2 - 2p_x P_x
\end{aligned}
\tag{31}
$$

When the momentum of the leptopion is in the scattering plane and the condition $\cos\theta = v_{\mathrm{cm}}/\beta$ is satisfied, U_1 behaves as $1/P_x$ and is singular only for forward scattering of the colliding nucleus.

Consider now the behavior of the diffraction term. The contribution of this term at the singularity is easy to evaluate since the dependence of the singular term on the impact parameter is exponential,

$$
\begin{aligned}
U_1 &\simeq (e^{Ab_{\mathrm{cr}}}/A + \mathrm{c.c})\ln(X)/2\\
A &= iP_x - m\gamma_1 \sin\psi/\beta\gamma\\
X &= [(1+s)^{1/2} + (1-s)^{1/2}]/[(1+s)^{1/2} - (1-s)^{1/2}]\\
s &= \sin\phi
\end{aligned}
\tag{32}
$$

To a good approximation, one can neglect the leptopion contribution in the expression for A and one obtains the following expression for the amplitude:

$$
\begin{aligned}
U_1 &\simeq [\sin(P_x b_{\mathrm{cr}})/P_x]\ln(X)/2\\
P_x &= M_R \beta \sin\alpha
\end{aligned}
\tag{33}
$$

Note that this term vanishes in the limit of the infinite impact parameter, as it should, since A_{id} is nonsingular.

In accordance with our expectations, the singular part of the production amplitude is indeed a rapidly oscillating function of β. Note that the period of the variation depends on the scattering angle α. This might explain why the peak has not been observed in some experiments (Cowan *et al.*, 1986): the averaging of the production over scattering angles smoothes out the peak.

The model predicts oscillatory behavior for the diffractive part of the production amplitude, but only one or two peaks have been observed and just at the critical velocity needed to overcome the Coulomb wall (Cowan *et al.*, 1986). This might be understood by the following argument.

(a) Finite-size effects need to be taken into account only for velocities for which nuclei classically have the possibility to touch each other. For smaller velocities one must use the quantum mechanical model with no cutoff in impact parameter and diffraction effects disappear.

(b) For velocities greater than the critical velocity, nuclear interactions come into play. As a consequence, collisions are not plane collisions anymore and elasticity is lost. This implies that the value of the momentum component P_x for the scattered nucleus is not determined by the scattering angle and collision velocity alone. One obtains a continuum of periods and the observed spectrum is averaged over different periodic spectra: no peaks are observed. Also, the average value of P_x gets smaller, with the consequence that the averaged production probability decreases.

5. BRIEF REVIEW OF TGD

I have shown that anomalous e^+e^- production might be understood as resulting from the decay of bound states of color-excited leptons. The idea of colored leptons does not fit quite naturally into the standard unification scenarios. In topological geometro dynamics (Pitkänen, 1981, 1983, 1985, 1986*a,b*, 1988) this idea arises quite naturally, so that it is perhaps worth reviewing briefly the basic ideas of this unification scenario in order to understand how the idea of colored leptons emerges in this scenario.

(a) Free particles correspond to 3-surfaces of some higher dimensional space $H = M^4 \times S$. The 3-surfaces can have boundary components and elementary fermions correspond to boundary components (generalizing the idea that quarks reside at the ends of the hadronic string). In this manner one obtains a topological explanation for family replication phenomenon: various boundary topologies (sphere, torus, etc.) correspond to various fermion families, so that only electroweak and color quantum numbers remain to be understood in terms of the geometry of $M^4 \times S$.

(b) The choice $H = M^4 \times CP_2$ makes it possible to understand the electroweak and color quantum numbers in terms of CP_2 geometry. Electroweak quantum numbers correspond to spin degrees of freedom of CP_2 and color quantum numbers to the isometries of CP_2. Baryon and lepton numbers correspond to different chiralities of H-spinors and B and L are exactly conserved quantum numbers unless chiral symmetry breaking occurs. In the spirit of the Skyrme model (Zahed and Brown, 1986), we can also identify electroweak, color, and gravitational fields as quantities related to the induced geometry of the 4-surfaces representing spacetime.

The isometry group of CP_2 is $SU(3)$ and is identified as the global color group. There are, however, strong reasons to expect that the central extension of local $SU(3)$ acts as the (at least approximate) symmetry group of the theory. The main motivation for this expectation is that the configuration space of the theory is union of the spaces Map(X, H), the space of maps from X to H, where X is a 3-manifold with arbitrary, possibly singular, manifold topology. Each of the spaces Map(X, H) can be regarded as a coset space G/H of two local gauge groups:

$$G = \mathrm{Map}(X, M^4 \times SU(3)), \qquad H = \mathrm{Map}(X, M^4 \times SU(2) \times U(1))$$

For ordinary finite-dimensional groups G and H the coset space G/H allows a G-invariant metric. If this holds true also in the local case, one expects that local $M^4 \times SU(3)$ or rather its central extension acts as the (at least approximate) symmetry group of the theory. Also, the analogy with string models (Green *et al.*, 1987) suggests strongly that physical states correspond to the central extension of this group, the Kac–Moody group (Kac, 1986).

Physical states are expected to lie in the infinite-dimensional representations of the 3-dimensional $SU(3)$ Kac–Moody group. The "vacuum states" of these representations correspond to multiplets of the ordinary color group and therefore to ordinary elementary particles. The excited states are obtained by applying $SU(3)$ Kac–Moody generators of these vacuum multiplets. Since Kac–Moody generators form a color octet, one obtains the triality rule: all states in representation have the triality of the ground-state multiplet. There are two possibilities to obtain colored leptons. They belong either to (a) Kac–Moody multiplets with a singlet ground state and can be regarded as vibrational excitations of ordinary leptons, or (b) the most naturally octet, ground state of a new Kac–Moody representation.

The naive expectation based on the properties of Kac–Moody representations in string models is that vibrational color excitations of leptons and quarks have masses of the order of the Planck mass. This would mean that colored light leptons more naturally correspond to the octet ground-state Kac–Moody representation. Of course, this does not exclude the possibility that light color confined states, leptomesons, of vibrational excitations of singlet ground states exist, although the excitations themselves have mass of order the Planck mass.

In the TGD picture leptomesons would correspond to 3-surfaces, perhaps stringlike objects (surfaces of type $X^2 \times S^2 \subset M^4 \times CP_2$, where X^2 is the minimal surface in M^4 and S^2 a geodesic sphere in CP_2), carrying leptonic quantum numbers and color excitation on the boundary components (ends of the stringlike object).

At this stage it is not possible to settle mathematically the question of whether this kind of state is indeed possible. There is, however, the intriguing possibility that heavy ion collisions make it possible to settle the question experimentally, contrary to the pessimistic belief that the nonpointlike nature of particles can be revealed at Planck energies only. A whole spectroscopy of leptohadrons would signal the nonpointlike nature of the fundamental objects!

6. WHY HAS LEPTONIC COLOR NOT BEEN OBSERVED?

The most obvious signal for leptonic color would be the existence of a sufficiently stable leptohadron. In order to explain the experimental absence of leptohadrons, one must assume that they are sufficiently heavy to decay to ordinary color singlet leptons so that leptohadrons become sufficiently short lived.

The fact that leptopions are bound states and not elementary particles implies that leptopion effects are not seen in anomalous magnetic moment of electrons (Itzykson and Zuber, 1980). In Babbha scattering (Itzykson and Zuber, 1980) no effects should be seen, since leptopions are not bound states of e^+e^-, but their color excitations. Note that the models which explain anomalous e^+e^- production as the decay of e^+e^- resonances in principle predict effects in Babbha scattering.

Strong interactions between leptons and ordinary hadrons (or ordinary leptons) would be a signal for leptonic color, but are expected to be absent. The reason is that gluons couple to each other only states in the same representation of a color group: gluons "measure" the color charge of the state in question. This means that gluon emission cannot transform leptons corresponding to different vacuum Kac–Moody color representations into each other. The result implies that ordinary leptons cannot interact strongly with hadrons or leptons.

Leptopion production in hadronic strong interactions is an obvious signal for leptonic color. Unless this production is prohibited by some selection rule, it dominates over the ordinary pion production, since the mass of the leptopion is so small. This kind of selection rule indeed exists.

Consider first leptopion production in perturbative QCD. The simplest manner to produce leptopions would be through gluon emission. A virtual gluon decays to two colored leptons; a second lepton emits a gluon and then combines with the second colored lepton to form a leptopion.

To simplify the situation, we can treat gluons as particles in initial and final states. This diagram is essentially identical to the diagram describing the decay of a pion to two photons when one expresses the leptopion field as the divergence of a leptonic axial current. For two-photon decay of a pion the amplitude is proportional to the hadronic contribution to the axial current anomaly and is nonvanishing by the asymmetry of electric charges with respect to weak isospin.

Now photons are only replaced with gluons and the amplitude is proportional to the contribution of gluons to the axial anomaly. Since gluonic coupling matrices are isospin symmetric (unlike photonic coupling matrices), this contribution vanishes. I believe that this result in fact holds to all orders in QCD perturbation theory.

A second manner to evaluate leptopion production is based on a nonperturbative approach using dual diagrams: in this approach production results as purely topological (splitting of a stringlike object). Now, however, the so-called OZI rule (Chew and Rosenzweig, 1976, 1978) forbids leptopion production: the OZI rule says that colored leptonic lines belonging to leptopions must originate from incoming or outcoming hadrons, which is impossible, since they do not contain colored leptons.

Thus, it seems that the electromagnetic anomaly is the only possible source for leptopions. There are two important constraints for the production of leptopions in strong electromagnetic fields. First, the scalar product $E \cdot B$ must be large. Far from the source region this scalar product tends to vanish: consider only the Coulomb field. Second, the region where $E \cdot B$ has considerable size cannot be too small compared with the leptopion de Broglie wavelength (large when compared with the size of nuclei, for example). If this condition does not hold, the plane wave appearing in the Fourier amplitude is essentially constant spatially and since the fields are approximately static, the Fourier component of $E \cdot B$ is expressible as a spatial divergence, which reduces to a surface integral over a surface far from the source region. The resulting amplitude is small, since fields in a far region have essentially vanishing $E \cdot B$.

So it seems that static electromagnetic fields do not produce leptopions. Collisions of nuclei seem to be especially favorable for leptopion production, since $E \cdot B$ is not only large, but produces a singularity in the production amplitude.

Of course, the presence of strong classical electromagnetic fields is not necessary for leptopion production. In the collisions of charged particles a virtual photon can decay into two colored leptons: colored lepton emits a photon before the combination with the second colored lepton to form a leptopion. In this case, however, the production rate is proportional to the third power of the fine structure constant. The hope is that this is enough to make production small enough.

APPENDIX. EVALUATION OF THE PRODUCTION AMPLITUDE

A1. General Form of the Integral

The amplitude for leptopion production with four-momentum

$$p=(p_0,\bar{p})=m\gamma_1(1,\, v\sin\theta\cos\phi,\, v\sin\theta\sin\phi,\, v\cos\theta)$$
$$\gamma_1=1/(1-v^2)^{1/2} \tag{A1}$$

is essentially the Fourier component of the instanton density

$$U(p)=\int e^{ip\cdot x}E\cdot B\,d^4x \tag{A2}$$

associated with the electromagnetic field of the colliding nuclei.

Coordinates are chosen so that the target nucleus is at rest at the origin of coordinates and the colliding nucleus moves along the positive z direction in the $y=0$ plane with velocity β. The orbit is approximated with a straight line with impact parameter b (Figure 1).

The instanton density is just the scalar product of the static electric field E of the target nucleus and the magnetic field B, the magnetic field associated with the colliding nucleus (Figure 2), which is obtained by boosting the Coulomb field of the static nucleus to velocity β. The flux lines of the magnetic field rotate around the direction of the velocity of the colliding nucleus so that the instanton density is indeed nonvanishing.

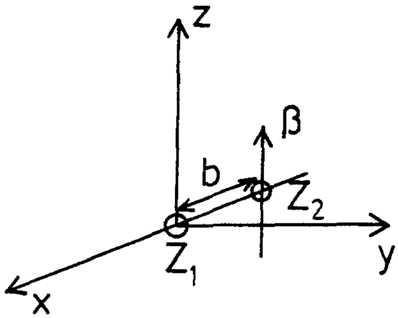

Fig. 1. The coordinates used to describe a nucleus–nucleus collision.

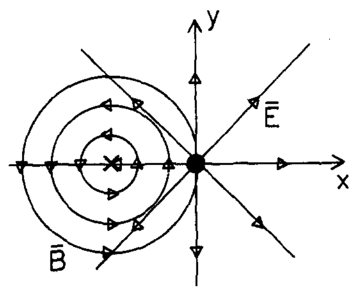

Fig. 2. Topology of the field lines for the electric field of the stationary nucleus and magnetic field of the colliding nucleus.

The Fourier transforms of E and B are given by the expressions

$$E_i(k) = \delta(k_0)k_i/k^2$$
$$B_i(k) = \delta(\gamma(k_0 - \beta k_z))k_j\varepsilon_{ijz}\, e^{ik_x b}/[(k_z/\gamma)^2 + k_T^2] \tag{A3}$$

The Fourier transform of the instanton density can be expressed as a convolution of the Fourier transforms of E and B,

$$U(p) = \int E(p-k)\cdot B(k)\, d^4k \tag{A4}$$

In the convolution the presence of two delta functions makes it possible to integrate over k_0 and k_z and the expression for U reduces to a twofold integral,

$$U(p) = \beta\gamma \int dk_x\, dk_y\, (k_x p_y - k_y p_x)/AB$$
$$A = (p_z - p_0/\beta)^2 + p_T^2 + k_T^2 - 2k_T\cdot p_T$$
$$B = k_T^2 + (p_0/\beta\gamma)^2 \tag{A5}$$
$$p_T = (p_x, p_y)$$

To carry out the remaining integrations, one can apply residue calculus.

(a) The k_y integral is expressed as a sum of two-pole contributions.

(b) The k_x integral is expressed as a sum of two-pole contributions plus two cut contributions.

A2. k_y Integration

Integration over k_y can be performed by completing the integration contour along the real axis to a half-circle in the upper half-plane (Figure 4).

The poles of the integrand come from the two factors A and B in the denominator and are given by the expressions

$$\begin{aligned} k_y^1 &= i[k_x^2 + (p_0/\beta\gamma)^2]^{1/2} \\ k_y^2 &= p_y + i[(p_z - p_0/\beta)^2 + p_x^2 + k_x^2 - 2p_xk_x]^{1/2} \end{aligned} \tag{A6}$$

One obtains for the amplitude an expression as a sum of two terms,

$$U = \int e^{ik_xb}(U_1 + U_2)\, dk_x \tag{A7}$$

corresponding to two poles in the upper half-plane.

The explicit expression for the first term is given by

$$\begin{aligned} U_1 &= \mathrm{RE}_1 + i\mathrm{IM}_1 \\ \mathrm{RE}_1 &= (k_xp_y^2 - p_x\,\mathrm{re}_1/2)/(\mathrm{re}_1^2 + \mathrm{im}_1^2) \\ \mathrm{IM}_1 &= (-k_xp_y\,\mathrm{re}_1/2K_1^{1/2} - p_xp_yK_1^{1/2})/(\mathrm{re}_1^2 + \mathrm{im}_1^2) \\ \mathrm{re}_1 &= (p_z - p_0/\beta)^2 + p_T^2 - (p_0/\beta\gamma)^2 - 2p_xk_x \\ \mathrm{im}_1 &= -2K_1^{1/2}p_y \\ K_1 &= k_x^2 + (p_0/\beta\gamma)^2 \end{aligned} \tag{A8}$$

The expression for the second term is given by

$$\begin{aligned} U_2 &= \mathrm{RE}_2 + i\mathrm{IM}_2 \\ \mathrm{RE}_2 &= -[(k_xp_y - p_xp_y)p_y + p_x\,\mathrm{re}_2/2]/(\mathrm{re}_2^2 + \mathrm{im}_2^2) \\ \mathrm{IM}_2 &= [-(k_xp_y - p_xp_y)\,\mathrm{re}_2/2K_2^{1/2} + p_xp_yK_2^{1/2}]/(\mathrm{re}_2^2 + \mathrm{im}_2^2) \\ \mathrm{re}_2 &= -(p_z - p_0/\beta)^2 + (p_0/\beta\gamma)^2 + 2p_xk_x + p_y^2 - p_x^2 \\ \mathrm{im}_2 &= 2p_yK_2^{1/2} \\ K_2 &= (p_z - p_0/\beta)^2 + p_x^2 + k_x^2 - 2p_xk_x \end{aligned} \tag{A9}$$

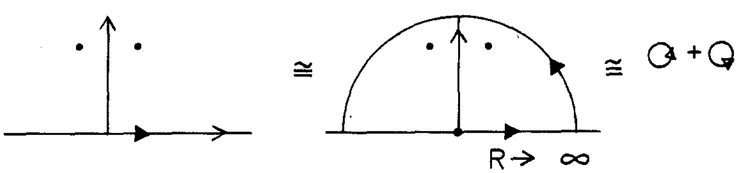

Fig. 3. Evaluation of k_y integral using residue calculus.

A3. k_x Integration

One cannot perform the k_x integration completely using the residue calculus. The reason is that the terms IM_1 and IM_2 have cuts in the complex plane. One can, however, reduce the integral to a sum of pole terms plus integrals over the cuts (Figure 4).

The poles of U_1 and U_2 come from the denominators and are in fact common for the two integrands. The explicit expressions for the poles in the upper half-plane where the integrand converges exponentially are given by

$$\begin{aligned} &\mathrm{re}_i^2+\mathrm{im}_i^2=0, \qquad i=1,2 \\ &k_x=[-b+i(-b^2+4ac)^{1/2}]/2a \\ &a=4p_T^2 \\ &b=-4[(p_z-p_0/\beta)^2+p_T^2-(p_0/\beta\gamma)^2]p_x \\ &c=[(p_z-p_0/\beta)^2+p_T^2-(p_0/\beta\gamma)^2]^2+4(p_0/\beta\gamma)^2p_y^2 \end{aligned} \tag{A10}$$

The cuts associated with U_1 and U_2 come from the square root terms K_1 and K_2. The condition for the appearence of the cut is that K_1 (K_2) is real and positive. In the case of K_1 this condition gives

$$k_x=it, \qquad t\in(0, p_0/\beta\gamma) \tag{A11}$$

In the case of K_2 the same condition gives

$$k_x=p_x+it, \qquad t\in(0, p_0/\beta-p_z) \tag{A12}$$

Both cuts are in the direction of the imaginary axis.

The integral over the real axis can be completed into an integral over a semicircle and this integral in turn can be expressed as a sum of three

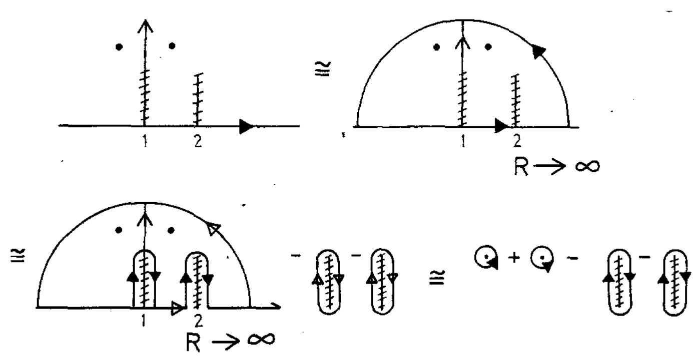

Fig. 4. Reduction of k_x integral to pole and cut terms using residue calculus.

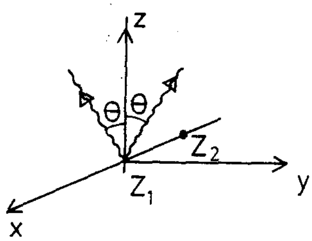

Fig. 5. Kinematically allowed configurations in the singularity of the production amplitude. The leptopion momentum is in the scattering plane and its velocity is a function of the scattering angle given by the condition $\cos\theta = 2v/[(1+v^2)\beta]$.

terms (Figure 5),

$$U = RES + CUT_1 + CUT_2 \tag{A13}$$

The first term corresponds to a contour which avoids the cuts and reduces to a sum of pole contributions. The second term corresponds to the addition of cut contributions.

In the following I give the expressions of various terms in the region $\phi \in [0, \pi/2]$. Using the symmetries

$$\begin{aligned} A(p_x, -p_y) &= -A(p_x, p_y) \\ A(-p_x, -p_y) &= \bar{A}(p_x, p_y) \end{aligned} \tag{A14}$$

of the amplitude, one can calculate the amplitude for other values of ϕ.

By a tedious but straightforward calculation one finds that the contribution of the poles to the amplitude is given by the expression

$$\begin{aligned} \text{RES} &= [\pi \sin\theta \cos\phi r/2(w^2 + r^2 \sin^2\theta)^{1/2}] \\ &\times e^{ikT}\{\sin\theta \sin\phi - i[1 + (k\beta\gamma)^2]^{1/2} \\ &+ [w^2 + (k - v\sin\theta\cos\phi)^2]^{1/2}\} \end{aligned} \tag{A15}$$

Here the definitions of various auxiliary variables are

$$\begin{aligned} w &= 1 - r\cos\theta \\ \gamma &= 1/(1-\beta^2)^{1/2} \\ r &= v_{\text{cm}}/\beta \\ v_{\text{cm}} &= 2v/(1+v^2) \\ T &= bm\gamma_1 \end{aligned} \tag{A16}$$

The integration variable for cuts is the imaginary part t of k_x. To get a more convenient form for cut integrals, one can perform a change of the integration variable

$$\begin{aligned} \cos\psi &= t/(p_0/\beta\gamma) \\ \cos\psi &= t/(p_0/\beta - p_z) \\ \psi &\in [0, \pi/2] \end{aligned} \tag{A17}$$

By a painstaking calculation one verifies that the expression for the contribution of the first cut is given by

$$\begin{aligned} \mathrm{CUT}_1 &= \sin\theta \sin\phi \int_0^{\pi/2} A\, d\psi/2 \\ A &= e^{-T\cos\psi}(\sin\theta\cos\phi + iK\cos\psi)/X_1 \\ X_1 &= \sin^2\theta(\sin^2\phi - \cos^2\psi) + K^2 - 2iK\sin\theta\cos\psi\cos\phi \\ K &= \beta\gamma(1 - v_{\mathrm{cm}}\cos\theta/\beta) \\ v_{\mathrm{cm}} &= 2v/(1+v^2) \\ T &= bm\gamma_1/\beta\gamma \end{aligned} \tag{A18}$$

The definitions of the various auxiliary variables are given in previous formulas.

The denominator X_1 vanishes when the conditions

$$\begin{aligned} \cos\theta &= \beta/v_{\mathrm{cm}} \\ \sin\phi &= \cos\psi \end{aligned} \tag{A19}$$

hold. In the forward direction the conditions express the vanishing of the z component of the leptopion velocity in the velocity cm frame, as one can easily realize by noticing that the conditions reduce to the condition $v = \beta/2$ in the nonrelativistic limit.

It turns out that the contribution of the first cut in fact diverges in the limit $\phi = 0$, which corresponds to the production of leptopions with momentum in the scattering plane and with direction angle $\cos\theta = \beta/v_{\mathrm{cm}}$.

The contribution of the second cut is given by the expression

$$\begin{aligned} \mathrm{CUT}_2 &= (u\sin\theta\sin\phi/2)e^{iTv\sin\theta\cos\phi} \int_0^{\pi/2} A\, d\psi \\ A &= e^{-Tu\cos\psi/\beta}[\sin\theta\cos\phi u \\ &\quad + i\cos\psi[w/v_{\mathrm{cm}} + (v/\beta)\sin^2\theta(\sin^2\phi - \cos^2\phi)]]/X_2 \end{aligned} \tag{A20}$$

The remaining auxiliary variables are defined as

$$\begin{aligned} X_2 = &\sin^2\theta[\sin^2\phi/\gamma^2 - u^2\cos^2\psi \\ &+ \beta^2(v^2\sin^2\theta - 2vw/v_{\rm cm})\cos^2\phi] + (w/v_{\rm cm})^2 \\ &+ 2iu\beta(v\sin^2\theta\cos\phi - w\cos\psi/v_{\rm cm})\sin\theta\cos\phi \end{aligned} \tag{A21}$$

$$u = 1 - \beta v\cos\theta$$

The denominator X_2 has no poles and the contribution of the second cut is therefore finite.

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